

Type II seesaw mechanism for Higgs doublets and the scale of new physics

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Abstract

We elaborate on an earlier proposal by Ernest Ma of a type II seesaw mechanism for suppressing the vacuum expectation values of some Higgs doublets. We emphasize that, by nesting this form of seesaw mechanism into various other seesaw mechanisms, one may obtain light neutrino masses in such a way that the new-physics scale present in the seesaw mechanism—the masses of scalar gauge- $SU(2)$ triplets, scalar $SU(2)$ doublets, or right-handed neutrinos—does not need to be higher than a few 10 TeV. We also investigate other usages of the type II seesaw mechanism for Higgs doublets. For instance, the suppression of the vacuum expectation values of Higgs doublets may realize Froggatt–Nielsen suppression factors in some entries of the fermion mass matrices.

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1 Introduction

The type I seesaw mechanism [1] is a favourite with high-energy physicists for explaining why the neutrino masses are so tiny. Unfortunately, in the usual realization of that mechanism the scale m_R of the Majorana masses of the right-handed neutrinos ν_R should be 10^{13} GeV (assuming that the natural scale of the neutrino Dirac mass matrix is the electroweak scale). As a consequence, the possibility of direct tests of the seesaw mechanism seems very remote. Lowering m_R to the TeV scale, although desirable from the point of view of experimental tests of the type I seesaw mechanism, apparently contradicts the aim with which it was invented, since it would require artificially suppressing the Yukawa couplings of the ν_R to values of order 10^{-5} .

Another mechanism for explaining the smallness of the neutrino masses is the type II seesaw mechanism [2], which suppresses the vacuum expectation values (VEVs) of the neutral components of scalar gauge- $SU(2)$ triplets, in such a way that the left-handed neutrinos ν_L , which acquire Majorana masses from their Yukawa couplings to those neutral components, are extremely light. Just like the type I seesaw mechanism, the type II seesaw mechanism requires a very high mass scale, which now occurs in the mass terms of the scalar triplets. Those large mass terms make the scalar triplets extremely heavy and therefore the type II seesaw mechanism, like the type I seesaw mechanism, is very difficult to test experimentally.

In the general case, for instance in Grand Unified Theories based on the gauge group $SO(10)$, both type I and type II seesaw mechanisms are present [3].

Several proposals have been made to bring the high mass scale of the seesaw mechanism(s) down to the TeV range, so that they might be experimentally testable, for instance at the Large Hadron Collider at CERN. The most straightforward possibility is to have cancellations within the type I seesaw mechanism such that m_R may be relatively low without the need to excessively suppress the Yukawa couplings [4]; the general conditions for this to happen were given in [5]. Cancellations between the type I and type II seesaw contributions to the neutrino masses have also been considered [6]. In the “inverse seesaw mechanism” [7] there is both a high scale in the TeV range and a low scale in the keV range. Other proposals include radiative neutrino masses generated by three-loop diagrams [8] or a specific type of mirror fermions [9].

In this letter we develop a proposal originally made in [10]. We elaborate on its two separate ideas:

- i. A type II seesaw mechanism suppresses the VEVs of some *Higgs doublets*.
- ii. A nesting of that type II seesaw mechanism *inside some other seesaw mechanism* (which may be of any type) allows one to lower the high mass scale of that seesaw mechanism.

The aim of this letter is to generalize the proposal of [10] in several directions:

- We describe (in section 2) the *general mechanism* for suppressing Higgs-doublet VEVs and give several examples thereof.

- We show (in section 3) that this “type II seesaw mechanism for Higgs doublets” may be nested *inside various seesaw mechanisms*.¹ We propose in particular a type II seesaw mechanism for Higgs doublets inside the usual type II seesaw mechanism for scalar triplets.
- We develop (in section 4) a multiply nested type II seesaw mechanism for many Higgs doublets which may mimic the Froggatt–Nielsen mechanism [12]. This suggests new ways of explaining the relative smallness of some charged-fermion masses—without the need for new heavy fermions as in the seesaw mechanism for Dirac fermions [13].

In summary, the message that we want to convey in this letter is that, by using the nesting of seesaw mechanisms, a heavy mass scale m_H many orders of magnitude larger than the electroweak scale $m_{\text{ew}} \sim 100 \text{ GeV}$ is not compelling; an m_H just two or three orders of magnitude above m_{ew} may suffice.

2 Type II seesaw mechanism for Higgs doublets

Consider a model with several Higgs doublets $\phi_j = (\phi_j^+, \phi_j^0)^T$, $j = 1, \dots, n_h$. The VEVs of the Higgs doublets are of the form

$$\langle \phi_j \rangle_0 = \begin{pmatrix} 0 \\ v_j \end{pmatrix}. \quad (1)$$

We assume that $|v_1| \sim m_{\text{ew}}$. Our aim is to produce a seesaw mechanism to suppress $|v_2|$. We write the scalar potential as

$$V = \sum_{j=1}^{n_h} \mu_j^2 \phi_j^\dagger \phi_j + (V_l + V_l^\dagger) + V_r. \quad (2)$$

We assume that $\mu_1^2 < 0$ and that $|\mu_1^2| \sim m_{\text{ew}}^2$ in order to generate a spontaneous symmetry breaking leading to $|v_1| \sim m_{\text{ew}}$. On the other hand, we assume that $\mu_2^2 > 0$ and that $\mu_2^2 = m_H^2 \gg m_{\text{ew}}^2$. In equation (2) V_l represents some terms linear in ϕ_2 which we assume to be present in V . All the remainder of V , *i.e.* everything but the mass terms for the Higgs doublets and the terms V_l and V_l^\dagger linear in ϕ_2 and ϕ_2^\dagger , respectively, is denoted V_r ; in the simplest cases V_r will consist only of quartic terms.

Inserting the VEVs into the potential one has

$$\langle V \rangle_0 = \sum_{j=1}^{n_h} \mu_j^2 |v_j|^2 + A v_2 + A^* v_2^* + \langle V_r \rangle_0, \quad (3)$$

where A has the dimension of the cube of a mass. Then, despite the positiveness of μ_2^2 , a non-vanishing VEV v_2 is induced, approximately given by

$$v_2 \approx -\frac{A^*}{\mu_2^2}. \quad (4)$$

¹The original proposal [10] was a type II seesaw mechanism for Higgs doublets within a type I seesaw mechanism. A later suggestion [11] was a type II seesaw mechanism for Higgs doublets within a type III seesaw mechanism.

The quantity A depends on the specific model. It has to contain at least one $v_j \neq v_2$ and this v_j will in general be of order m_{ew} . If we assume that μ_2^2 is the only parameter in the scalar potential of order m_H^2 , then we expect $|A| \sim m_{\text{ew}}^3$. In this case $|v_2| \sim m_{\text{ew}}^3/m_H^2$ is suppressed by *two* powers of m_{ew} over m_H , where m_H is the scale of new physics.

Two Higgs doublets and a softly broken symmetry: In the original proposal [10] of the type II seesaw mechanism for Higgs doublets there were only two Higgs doublets and no other scalar multiplets.² A $U(1)$ symmetry

$$\phi_2 \rightarrow e^{i\alpha} \phi_2 \quad (5)$$

was softly broken in the scalar potential by

$$V_l = \mu^2 \phi_1^\dagger \phi_2. \quad (6)$$

Then,

$$v_2 \approx -\frac{\mu^{2*} v_1}{\mu_2^2}. \quad (7)$$

The VEV v_1 alone must produce the W^\pm and Z^0 masses, therefore $|v_1| \approx 174 \text{ GeV} \sim m_{\text{ew}}$. We assume that $\mu_2^2 = m_H^2 \gg m_{\text{ew}}^2$ and that $|\mu^2| \lesssim m_H^2$, where the symbol \lesssim means “not much larger than”. We may assume that $|\mu^2| \sim m_{\text{ew}}^2$ and then v_2 is suppressed by two powers of m_{ew}/m_H relative to v_1 .³

General two-Higgs-doublet model: Actually, one could dispense with any symmetry and consider the general two-Higgs-doublet model, employing the same assumptions as in the previous paragraph. Then in V_l not only the term of equation (6) is present but also

$$\left(\phi_1^\dagger \phi_1\right) \left(\phi_1^\dagger \phi_2\right). \quad (8)$$

Therefore, one has two sources which induce a non-zero v_2 . As discussed in [15], one obtains a suppression factor of v_2 of the same order of magnitude as before.

Two Higgs doublets and a scalar singlet: If we dislike soft symmetry breaking the simplest alternative is to introduce into the theory a complex scalar gauge singlet χ with VEV v_χ . The $U(1)$ symmetry (5) becomes

$$\phi_2 \rightarrow e^{i\alpha} \phi_2, \quad \chi \rightarrow e^{-i\alpha} \chi. \quad (9)$$

Then,

$$V_l = m \phi_1^\dagger \phi_2 \chi, \quad (10)$$

$$v_2 \approx -\frac{m^* v_\chi^* v_1}{\mu_2^2}. \quad (11)$$

There is a large degree of arbitrariness in the orders of magnitude of $|m|$ and of the VEV of χ , but we may conservatively assume them to be of order m_{ew} . Then once again $|v_2/v_1| \sim (m_{\text{ew}}/m_H)^2$.

²A related scenario with the assumption $\mu_1^2 = 0$ was proposed in [14].

³As a matter of fact, since V_l in this case breaks softly the symmetry (5), it would be technically natural to assume $|\mu^2| \ll m_{\text{ew}}^2$, as was done in [10], and then $|v_2|$ would be even smaller.

Symmetry \mathbb{Z}_2 instead of $U(1)$: Instead of the $U(1)$ symmetry (5) originally used in [10] one may employ the weaker symmetry

$$\phi_2 \rightarrow -\phi_2, \quad \chi \rightarrow -\chi. \quad (12)$$

In this case χ may as a matter of fact be a real field. The symmetry (12) allows for a richer scalar potential, with extra terms $(\phi_1^\dagger \phi_2)^2$ and χ^4 and their Hermitian conjugates.

Three Higgs doublets: A more complicated model has three Higgs doublets and a symmetry.

$$\mathbb{Z}_4 : \quad \phi_2 \rightarrow -\phi_2, \quad \phi_3 \rightarrow i\phi_3. \quad (13)$$

Note that we now assume $\mu_j^2 < 0$ and $|\mu_j^2| \sim m_{\text{ew}}^2$ for both $j = 1, 3$. Then

$$V_l = \lambda \left(\phi_3^\dagger \phi_2 \right) \left(\phi_3^\dagger \phi_1 \right), \quad (14)$$

$$V_r = \lambda' \left(\phi_1^\dagger \phi_2 \right)^2 + \dots, \quad (15)$$

$$v_2 \approx -\frac{\lambda^* v_1^* v_3^2}{\mu_2^2}. \quad (16)$$

In this case $|v_1|$ and $|v_3|$ are *necessarily* of order m_{ew} (or smaller) and one needs no extra assumption to conclude that $|v_2| \sim m_{\text{ew}}^3/m_H^2$.

Before we proceed to investigate the nesting of seesaw mechanisms, we want to mention some simple applications of a seesaw mechanism for Higgs doublets. Suppose that ϕ_2 has Yukawa couplings only to the ν_R , and ϕ_1 to all charged fermionic gauge- $SU(2)$ singlets. Then with the small VEV v_2 we have the option of a seesaw mechanism for *Dirac* neutrinos, if we dispense with a ν_R Majorana mass term. We would then use $|v_2| \sim m_{\text{ew}}^3/m_H^2 \sim 1 \text{ eV}$, where we assume 1 eV to be the scale of the light-neutrino masses, obtaining the estimate $m_H \sim \sqrt{10^{33}} \text{ eV} = 10^{7.5} \text{ GeV}$. We could also try to “explain” the smallness of the down-type-quark masses as compared to the up-type-quark masses by enforcing the coupling of ϕ_1 to the up-quark singlets and ϕ_2 to the down-quark singlets in the Yukawa couplings. Assuming

$$\left| \frac{v_2}{v_1} \right| \sim \frac{m_b}{m_t} \sim \left(\frac{m_{\text{ew}}}{m_H} \right)^2, \quad (17)$$

we find for the mass of the heavy Higgs doublet $m_H \sim 6 m_{\text{ew}}$.

3 Nesting of seesaw mechanisms

3.1 Type I seesaw mechanism

The type I seesaw formula is [1]

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D, \quad (18)$$

where \mathcal{M}_ν is the effective ν_L Majorana mass matrix, M_R is the Majorana mass matrix of the ν_R and M_D is the Dirac mass matrix connecting the ν_R to the ν_L . This Dirac mass matrix is generated by Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = \bar{\nu}_R \tilde{\phi}_2^\dagger Y D_L + \text{H.c.}, \quad (19)$$

where Y is a matrix (in flavour space) of Yukawa coupling constants, $D_L = (\nu_L, \ell_L)^T$ are $SU(2)$ doublets of left-handed leptons, ϕ_2 is the Higgs doublet whose VEV is suppressed by a type II seesaw mechanism and $\tilde{\phi}_2 \equiv i\tau_2 \phi_2^*$.

In order for the Yukawa couplings of the ν_R in equation (19) to involve only the Higgs doublet ϕ_2 , one needs to suitably extend the symmetries $U(1)$, \mathbb{Z}_2 or \mathbb{Z}_4 of the previous section. In the case of the $U(1)$ symmetry, one must add $D_L \rightarrow e^{-i\alpha} D_L$ and $\ell_R \rightarrow e^{-i\alpha} \ell_R$ to the assignment (9) (the ℓ_R are the right-handed charged-lepton singlets) [10]. In the case of the \mathbb{Z}_2 or \mathbb{Z}_4 symmetries, one must add $\nu_R \rightarrow -\nu_R$ to the assignments (12) and (13), respectively.

It follows from equation (19) that $M_D = v_2 Y$, hence $\mathcal{M}_\nu = -v_2^2 Y^T M_R^{-1} Y$. As before, we assume the matrix elements of \mathcal{M}_ν to be of order eV. If we allowed the VEV v_2 to be of order the electroweak scale $m_{\text{ew}} \sim 100 \text{ GeV}$, and assuming the Yukawa coupling constants to be of order unity, we would find the scale m_R of M_R to be of order 10^{13} GeV , as advertised in the introduction. Lowering m_R to the TeV scale while keeping $v_2 \sim m_{\text{ew}}$ requires (assuming no cancellation mechanism) the Yukawa couplings to be of order 10^{-5} , as also advertised in the introduction. But if $v_2 \sim m_{\text{ew}}^3/m_H^2$ is suppressed by a type II seesaw mechanism for Higgs doublets, as first proposed in [10], then $1 \text{ eV} \sim m_{\text{ew}}^6/(m_R m_H^4)$ even with Yukawa coupling constants of order unity. This represents a *fivefold suppression* of the neutrino masses. Assuming for simplicity $m_R = m_H$, one obtains

$$m_H \sim \sqrt[5]{10^{66}} \text{ eV} \approx 16 \text{ TeV}. \quad (20)$$

3.2 Type II seesaw mechanism

In the type II seesaw mechanism [2], a scalar gauge- $SU(2)$ triplet

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \quad (21)$$

is introduced such that the ν_L acquire Majorana masses through the VEV of the neutral component of the scalar triplet:

$$\langle \Delta \rangle_0 = \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}. \quad (22)$$

This VEV is induced by the term linear in Δ in the scalar potential and is suppressed by the high mass of the scalar triplet.

In order for the terms linear in Δ to involve only the Higgs doublet ϕ_2 whose VEV is suppressed by the type II seesaw mechanism discussed in section 2, we introduce a \mathbb{Z}_4

symmetry⁴:

$$\phi_2 \rightarrow i\phi_2, \quad \Delta \rightarrow -\Delta. \quad (23)$$

We write the scalar potential as

$$V = \sum_{j=1}^2 \mu_j^2 \phi_j^\dagger \phi_j + \mu_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \left(\mu^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right) + \left(\mu' \phi_2^\dagger \Delta \tilde{\phi}_2 + \text{H.c.} \right) + V_q, \quad (24)$$

where V_q consists only of quartic terms. The \mathbb{Z}_4 symmetry is softly broken by operators of dimension two. Instead of a softly broken symmetry for the type II seesaw mechanism for the VEV of ϕ_2 , one could employ one of the alternatives given in section 2. In order to have a Dirac mass term for charged leptons and a Majorana mass term for ν_L generated by VEVs of ϕ_1 and Δ , respectively, one needs to extend the \mathbb{Z}_4 symmetry in equation (23) to $D_L \rightarrow iD_L$ and $\ell_R \rightarrow i\ell_R$.

Now we proceed according to section 2. On the one hand, we assume that $\mu_1^2 < 0$ and that $|\mu_1^2| \sim m_{\text{ew}}^2$ in order to generate a spontaneous symmetry breaking with $|v_1| \sim m_{\text{ew}}$. On the other hand, we require

$$\mu_2^2 > 0, \quad \mu_\Delta^2 > 0 \quad \text{and} \quad \mu_2^2 \sim \mu_\Delta^2 \sim m_H^2 \gg m_{\text{ew}}^2. \quad (25)$$

The terms linear in ϕ_2 and Δ in equation (24) generate non-vanishing VEVs v_2 and v_Δ , respectively. Using the result for v_2 of equation (7), the VEV of Δ is given by

$$v_\Delta \approx -\frac{\mu'^* v_2^2}{\mu_\Delta^2} \approx -\frac{\mu'^* (\mu^{2*})^2 v_1^2}{\mu_2^4 \mu_\Delta^2}. \quad (26)$$

As before, there is a degree of arbitrariness in the orders of magnitude of $|\mu|$ and $|\mu'|$, but we may assume them to be of order m_{ew} . Then v_Δ is suppressed by six powers of m_{ew}/m_H relative to v_1 . Keeping the Yukawa coupling constants of order unity, this represents a *sixfold suppression* of the neutrino masses. Assuming again the matrix elements of \mathcal{M}_ν to be of order eV which amounts to $v_\Delta \sim 1$ eV, with equation (25) we estimate

$$m_H \sim \sqrt[6]{10^{77}} \text{ eV} \approx 7 \text{ TeV}. \quad (27)$$

By raising the mass of the ϕ_2 to 20 TeV, one shifts μ_Δ below 1 TeV, and the δ^{++} , whose mass is just μ_Δ , could possibly be within reach of the LHC—see for instance [17] and the references therein.

4 Multiple nesting of type II seesaw mechanisms for Higgs doublets

In this section we show that a multiple nesting of successive type II seesaw mechanisms for several Higgs doublets is able to realize Froggatt–Nielsen [12] suppression factors by using only Higgs doublets and renormalizable interactions.

⁴In [16] a softly broken $U(1)$ symmetry has been used instead, together with assumptions on the soft-breaking parameters in the scalar potential.

As an example, we consider the hierarchy of charged-fermion masses:

$$\begin{aligned}
m_t &\sim m_{\text{ew}} \\
m_b, m_c, m_\tau &\sim 2 \text{ GeV}, \\
m_s, m_\mu &\sim 0.1 \text{ GeV}, \\
m_u, m_d &\sim 0.005 \text{ GeV}, \\
m_e &\sim 0.0005 \text{ GeV}.
\end{aligned} \tag{28}$$

This hierarchy suggests that the charged-fermion mass matrices may involve a suppression factor $\epsilon \sim 1/20$ according to the pattern

$$M_u \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ 1 & \epsilon & \epsilon^3 \\ 1 & \epsilon & \epsilon^3 \end{pmatrix}, \quad M_d \sim \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon & \epsilon^2 & \epsilon^3 \end{pmatrix}, \quad M_\ell \sim \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^4 \\ \epsilon & \epsilon^2 & \epsilon^4 \\ \epsilon & \epsilon^2 & \epsilon^4 \end{pmatrix}. \tag{29}$$

The suppression factors in the various elements of these mass matrices may be explained *à la* Froggatt–Nielsen [12] as the result of a spontaneously broken horizontal symmetry. We suggest to view them instead as the product of a nested type II seesaw mechanism for Higgs doublets.⁵

We postulate the existence of six Higgs doublets $\phi_{1,\dots,6}$, where ϕ_1 and ϕ_2 have VEVs of order m_{ew} and Yukawa couplings which generate the first column of M_u , ϕ_3 has VEV of order ϵm_{ew} and generates the second column of M_u and the first columns of M_d and M_ℓ , ϕ_4 has VEV of order $\epsilon^2 m_{\text{ew}}$ and its Yukawa couplings yield the second columns of M_d and M_ℓ , and so on.

We implement the hierarchy of VEVs in the following way. The scalar potential is of the form

$$V = \sum_{j=1}^6 \left(\mu_j^2 + \frac{\lambda_j}{2} \phi_j^\dagger \phi_j \right) \phi_j^\dagger \phi_j + \sum_{j < k} \left(\lambda'_{jk} \phi_j^\dagger \phi_j \phi_k^\dagger \phi_k + \lambda''_{jk} \phi_j^\dagger \phi_k \phi_k^\dagger \phi_j \right) + V_t + V_t^\dagger. \tag{30}$$

We assume that μ_1^2 and μ_2^2 are both negative and of order m_{ew}^2 , while $\mu_{3,\dots,6}^2$ are positive and of order m_H^2 , with $(m_{\text{ew}}/m_H)^2 \sim \epsilon$.⁶ The VEV of ϕ_3 is induced out of the VEVs of ϕ_1 and ϕ_2 via a term

$$\kappa_1 \phi_1^\dagger \phi_2 \phi_1^\dagger \phi_3 \tag{31}$$

in V_t . This leads to $v_3 \approx -\kappa_1^* v_1^2 v_2^*/\mu_3^2$. Since the coupling constant $|\kappa_1| \lesssim 1$, $|v_3|$ is of order m_{ew}^3/m_H^2 . Afterwards the VEV of ϕ_4 is induced by a further term in V_t ,

$$\kappa_2 \phi_2^\dagger \phi_3 \phi_2^\dagger \phi_4. \tag{32}$$

This leads to $v_4 \approx -\kappa_2^* v_2^2 v_3^*/\mu_4^2 \sim m_{\text{ew}}^5/m_H^4$. The VEVs v_5 and v_6 are successively induced by terms

$$\kappa_3 \phi_1^\dagger \phi_4 \phi_1^\dagger \phi_5, \tag{33}$$

$$\kappa_4 \phi_2^\dagger \phi_5 \phi_2^\dagger \phi_6, \tag{34}$$

⁵A similar idea was already put forward in [18] and subsequently combined with the leptonic model of [10] in a supersymmetric way [19].

⁶With $\epsilon \sim 1/20$ this produces only a slight difference between m_{ew} and m_H . This certainly constitutes a drawback of the present model.

respectively, in V_t .⁷

In order to make sure that there are in V_t no other terms which might induce larger (unsuppressed) VEVs, we must impose a symmetry \mathcal{S} on the theory. For simplicity we assume that symmetry to be Abelian:

$$\mathcal{S}: \quad \phi_j \rightarrow \sigma_j \phi_j, \quad (35)$$

with $|\sigma_j| = 1$ for $j = 1, \dots, 6$. We assume, of course, the six factors $\sigma_{1,\dots,6}$ to be all different. In order for the four terms (31)–(34) to be allowed, we must assume

$$\sigma_1^2 = \sigma_2 \sigma_3, \quad \sigma_2^2 = \sigma_3 \sigma_4, \quad \sigma_1^2 = \sigma_4 \sigma_5, \quad \sigma_2^2 = \sigma_5 \sigma_6. \quad (36)$$

Therefore,

$$\sigma_3 = \frac{\sigma_1^2}{\sigma_2}, \quad \sigma_4 = \frac{\sigma_2^3}{\sigma_1^2}, \quad \sigma_5 = \frac{\sigma_1^4}{\sigma_2^3}, \quad \sigma_6 = \frac{\sigma_2^5}{\sigma_1^4}. \quad (37)$$

It follows that the bilinears $\phi_j^\dagger \phi_k$ ($j < k$) transform as

$$\begin{aligned} \phi_1^\dagger \phi_2 &: \frac{\sigma_2}{\sigma_1}, \\ \phi_1^\dagger \phi_3 &: \frac{\sigma_1}{\sigma_2}, \quad \phi_2^\dagger \phi_3 : \frac{\sigma_2^2}{\sigma_1^2}, \\ \phi_1^\dagger \phi_4 &: \frac{\sigma_2^3}{\sigma_1^3}, \quad \phi_2^\dagger \phi_4 : \frac{\sigma_2^2}{\sigma_1^2}, \quad \phi_3^\dagger \phi_4 : \frac{\sigma_2^4}{\sigma_1^4}, \\ \phi_1^\dagger \phi_5 &: \frac{\sigma_1^3}{\sigma_2^3}, \quad \phi_2^\dagger \phi_5 : \frac{\sigma_1^4}{\sigma_2^4}, \quad \phi_3^\dagger \phi_5 : \frac{\sigma_1^2}{\sigma_2^2}, \quad \phi_4^\dagger \phi_5 : \frac{\sigma_1^6}{\sigma_2^6}, \\ \phi_1^\dagger \phi_6 &: \frac{\sigma_2^5}{\sigma_1^5}, \quad \phi_2^\dagger \phi_6 : \frac{\sigma_2^4}{\sigma_1^4}, \quad \phi_3^\dagger \phi_6 : \frac{\sigma_2^2}{\sigma_1^2}, \quad \phi_4^\dagger \phi_6 : \frac{\sigma_2^2}{\sigma_1^2}, \quad \phi_5^\dagger \phi_6 : \frac{\sigma_2^8}{\sigma_1^8}. \end{aligned} \quad (38)$$

We assume that all the factors in this list are different from unity—else there would be (at least) two Higgs doublets transforming identically under \mathcal{S} —and also different from each other—so that there are as few terms as possible in V_t . This requires

$$\sigma_1^p \neq \sigma_2^p \text{ for } p = 1, 2, \dots, 14. \quad (39)$$

Therefore we must choose for \mathcal{S} a group \mathbb{Z}_n with $n > 14$. It is enough to choose $\mathcal{S} = \mathbb{Z}_{15}$ with

$$\phi_1 \rightarrow \omega \phi_1, \quad \phi_2 \rightarrow \omega^2 \phi_2, \quad \phi_3 \rightarrow \phi_3, \quad \phi_4 \rightarrow \omega^4 \phi_4, \quad \phi_5 \rightarrow \omega^{13} \phi_5, \quad \phi_6 \rightarrow \omega^6 \phi_6, \quad (40)$$

where $\omega \equiv \exp(2i\pi/15)$. From the list (38) we learn that the full V_t is

$$\begin{aligned} V_t = & \kappa_1 \phi_1^\dagger \phi_2 \phi_1^\dagger \phi_3 + \kappa_2 \phi_2^\dagger \phi_3 \phi_2^\dagger \phi_4 + \kappa_3 \phi_1^\dagger \phi_4 \phi_1^\dagger \phi_5 + \kappa_4 \phi_2^\dagger \phi_5 \phi_2^\dagger \phi_6 \\ & + \kappa_5 \phi_3^\dagger \phi_2 \phi_3^\dagger \phi_5 + \kappa_6 \phi_4^\dagger \phi_2 \phi_4^\dagger \phi_6 + \kappa_7 \phi_3^\dagger \phi_6 \phi_4^\dagger \phi_5 + \kappa_7' \phi_3^\dagger \phi_5 \phi_4^\dagger \phi_6 \\ & + \kappa_8 \phi_2^\dagger \phi_6 \phi_4^\dagger \phi_3 + \kappa_8' \phi_2^\dagger \phi_3 \phi_4^\dagger \phi_6 + \kappa_9 \phi_2^\dagger \phi_5 \phi_3^\dagger \phi_4 + \kappa_9' \phi_2^\dagger \phi_4 \phi_3^\dagger \phi_5. \end{aligned} \quad (41)$$

⁷Instead of the terms (31)–(34) we might imagine other possibilities. The present text thus constitutes only a proof of the viability of the mechanism.

It is easy to check that with this V_t VEVs with the right powers of the suppression factor $\epsilon \sim (m_{\text{ew}}/m_H)^2$ are generated. One obtains

$$v_3 \approx -\kappa_1^* \frac{v_1^2 v_2^*}{\mu_3^2}, \quad (42)$$

$$v_4 \approx -\kappa_2^* \frac{v_2^2 v_3^*}{\mu_4^2}, \quad (43)$$

$$v_5 \approx -\kappa_3^* \frac{v_1^2 v_4^*}{\mu_5^2} - \kappa_5^* \frac{v_3^2 v_2^*}{\mu_5^2}, \quad (44)$$

$$v_6 \approx -\kappa_4^* \frac{v_2^2 v_5^*}{\mu_6^2} - (\kappa_8^* + \kappa_8'^*) \frac{v_2 v_4 v_3^*}{\mu_6^2}. \quad (45)$$

The other terms in V_t generate subdominant (in terms of ϵ) contributions to the VEVs.

5 Conclusions

The main point in this letter is the observation that the VEVs of some Higgs doublets may be suppressed by a type II seesaw mechanism in the same way as the VEVs of scalar gauge triplets. We have furthermore emphasized that this Higgs-doublet type II seesaw mechanism may be combined with other seesaw mechanisms of any type—I, II, III or even with itself in a multiply nested way. If there are only two mass scales at our disposal, the electroweak scale m_{ew} and a heavy scale $m_H \gg m_{\text{ew}}$, one may through this procedure suppress some mass terms by a factor $(m_{\text{ew}}/m_H)^p$, where the power p can be considerably larger than 1 as in the standard type I seesaw case. While the standard seesaw mechanisms are applied to Majorana neutrinos, the type II seesaw mechanism for Higgs doublets, whether in its simple or in its multiply nested form, is able to suppress any Dirac-fermion masses without one having to introduce any new fermionic degrees of freedom in the theory.

Our aim was not to promote a specific type of seesaw mechanism, rather to point out the wealth of possible scenarios. It is also beyond the scope of this letter to check the compatibility of each particular scenario with the experimental data, for instance with electroweak precision tests. Thus, in individual cases the parameter space may have to be restricted or the scenario modified.

A seesaw mechanism always involves the *ad hoc* introduction of a heavy scale m_H . The usual belief is that either the new physics at m_H is not directly accessible by experiment because that scale is too high, or contrived cancellation mechanisms are needed to lower m_H . The main message of this letter is that neither of the two conclusions is compelling. As originally demonstrated in a specific case [10] and generalized in this letter, the nesting of the type II seesaw mechanism for Higgs doublets with other seesaw mechanisms, or with itself, provides a very simple method to lower m_H . This method requires an extension of the scalar sector and, therefore, leads to new physics at the scale m_H .

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